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Target tracking using artificial potentials and sliding mode control

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In this article we develop an algorithm for capturing/intercepting a moving target based on the sliding mode control method. First, we consider a "kinematic" model (in a sense) for the capture/intercept problem and develop a method for that case. Then, we build on the developed method to include general fully actuated vehicle dynamics for the pursuer agent. The algorithm is robust with respect to the system uncertainties and additive disturbances. Finally, we also provide a numerical simulation in order to illustrate the procedure.

1. Introduction

In recent years, there has been increasing attention and effort by the controls community on importing biological principles into the controls literature and developing biologically inspired systems. These include developing autonomous agents (either single or multiple) performing complex tasks. The motivation is that many biological systems have designs very well adapted to their environments (tuned by the evolutionary process for millions of years), hence there might be useful principles that engineers can learn and use in developing engineering systems. However, this is best accomplished within the framework of systems perspective and its well established, rigorous methods developed through years of experience.

In nature, the survival of many species may critically depend on their ability to capture a prey (a target) or escape capture from a predator (a pursuer). In this article we develop a method for intercepting/capturing (or simply tracking) a moving target using potential functions and the sliding mode control technique. The sliding mode control method is an important technique that has been used extensively for robot navigation and control (we will not mention these here). It has a variety of attractive properties, including its robustness to system uncertainties and external disturbances and its ability to reduce the problem of controller design to a lower dimension with the choice of an appropriate switching surface. See Utkin (1977), Decarlo *et al.* (1988) and Young *et al.* (1999) and references therein for a short introduction to sliding mode control and Utkin (1992) for more detailed discussions. Similarly, the articles in Drakunov (1992), Drakunov and Utkin (1995) and Haskara *et al.* (1998) describe how the sliding mode control method can be used for developing state observers.

In Utkin *et al.* (1991) and Gulgner and Utkin (1993, 1995) the sliding mode control technique was used for robot navigation and obstacle avoidance in an environment modeled with harmonic potentials. The strategies there are based on forcing the motion of the robot along the gradient of an artificial potential field, which represents the environment. In particular, it was created by placing positive charges at the obstacle positions and negative charge at the goal point. Similarly, in Gazi (2005) it was shown that this method can be used for implementing aggregating swarms as well as formation control. In this case, the potential function included or

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modeled also the interactions between the members of the swarm (group). The results in Gazi (2005) constitute a possible implementation method of earlier results developed in Gazi and Passimo (2003, 2004a, b).

In Massoud and Bayoumi (1994) and Massoud (1995) the authors describe a method for target intercepting based on harmonic artificial potentials – an approach that is a generalization of the harmonic potential fields approach used for stationary targets (such as those in Utkin *et al.* (1991) and Guldner and Utkin (1993, 1995)). They employ a time dependent potential field, which is generated using the linear wave equation. Despite some of their shortcomings, these articles constituted a motivation for this work.

This paper is organized as follows. In the next section we discuss a method for intercepting a maneuvering target using, in a sense, a "kinematic" model for the pursuer (much like those considered in Massoud and Bayoumi (1994) and Massoud (1995)). For this model we develop an algorithm based on the sliding mode control method. In § 3, we consider a general fully actuated dynamic model of the pursuer (much like those considered in Utkin *et al.* (1991), Guldner and Utkin (1993, 1995) and Gazi (2005)) and build on the results in § 2. The developed method is once more based on the sliding mode control strategy. A key idea for the method is to use a low pass filter (much like is done in sliding mode observers (Drakunov 1992, Drakunov and Utkin 1995, Haskara *et al.* 1998) in order to smooth the switching term from the previous stage of the controller design (i.e., the one in § 2). In § 4 we provide illustrative numerical simulation examples, and in § 5 we conclude with a few remarks.

2. Potential functions based "kinematic" model for target tracking

In this section we consider the problem of a pursuer tracking a target in an n-dimensional Euclidean space. Let the position of the (possibly moving) target (to be tracked or intercepted) be denoted by x_t and the position of the pursuer be denoted by x_p. Moreover, assume that the pursuer moves based on the equation

$$\dot{x}_p = g(x_p, x_t), \tag{1}$$

where $g: \mathbb{R}^{2n} \to \mathbb{R}^n$ represents its motion dynamics. The objective is to design $g(x_p, x_t)$ such that

$$\lim_{t\to\infty} \|x_p - x_t\| = 0. \tag{2}$$

With this objective, we define $J(x_p, x_t)$ as the potential of the distance between the target and the pursuer and choose it such that

- it has its unique minimum at $x_p = x_t$.
- it satisfies

$$\nabla_{x_p} J(x_p, x_t) = -\nabla_{x_t} J(x_p, x_t). \tag{3}$$

Note that the functions which are functions of $\|x_p - x_t\|$ satisfy this assumption. In fact, one possible function which satisfies these requirements is

$$J(x_p, x_t) = \frac{1}{2}\|x_p - x_t\|^2. \tag{4}$$

Potential functions have been used extensively for robot navigation and control (Khatib 1986, Rimon and Koditschek 1992). It might be possible to use a variety of different potential functions here. For example, one option could be the use of harmonic potentials such as considered in Massoud and Bayoumi (1994) and Massoud (1995). In the rest of this article we will use the potential in (4) although other potentials are also possible.

In order to be able to guarantee satisfaction of the objective in (2), we need the potential $J(\cdot,\cdot)$ to be a decreasing function of time. Its time derivative is given by

$$\dot{J} = \nabla_{x_p} J^\top(x_p, x_t)\dot{x}_p + \nabla_{x_t} J^\top(x_p, x_t)\dot{x}_t.$$

Then, since $J(x_p, x_t)$ satisfies the condition in (3), its derivative can be written as

$$\dot{J} = \nabla_{x_p} J^\top(x_p, x_t)(\dot{x}_p - \dot{x}_t). \tag{5}$$

If x_t and $\dot{x}_t$ were known, then one could choose

$$\dot{x}_p = g(x_p, x_t) = \dot{x}_t - \alpha\nabla_{x_p} J(x_p, x_t), \tag{6}$$

for some constant $\alpha > 0$ leading to the equality

$$\dot{J} = -\alpha\left\|\nabla_{x_p} J(x_p, x_t)\right\|^2.$$

However, assuming that both x_t and $\dot{x}_t$ are known is a strong (i.e., restrictive) assumption since usually it is not possible for the pursuer to know both the current position and velocity of the target. Below we will consider two cases. First, we will assume that the position x_t is known and will develop the controller accordingly. Later, we will show that it is possible to design a tracking controller even if only the sign of $\nabla_{x_p} J(x_p, x_t)$ is known.

Assumption 1: *The position x_t of the moving target is known. Moreover, its velocity satisfies $\|\dot{x}_t\| \leq \gamma_t$ for some known $\gamma_t > 0$.*

The assumption that x_t is known is not a weak assumption. However, for now we will stick with this assumption and later we will show how it can be relaxed to the case in which only the sign of $\nabla_{x_p} J(x_p, x_t)$ is known. The assumption that $\|\dot{x}_t\| \leq \gamma_t$ constitutes a realistic assumption since any realistic agent has a bounded velocity.

With Assumption 1 one can choose the pursuer dynamics $g(x_p, x_t)$ as

$$\dot{x}_p = -\alpha \nabla_{x_p} J(x_p, x_t) - \beta \,\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big), \qquad (7)$$

where the constant parameters α and β are chosen as $\alpha > 0$ and $\beta \geq \gamma_t$ and $\mathrm{sign}(\cdot)$ is the signum function defined as

$$\mathrm{sign}(y) = \begin{cases} 1 & \text{if } y > 0, \\ 0 & \text{if } y = 0, \\ -1 & \text{if } y < 0, \end{cases}$$

for a scalar $y \in \mathbb{R}$ and operated elementwise for a vector $y \in \mathbb{R}^n$, i.e., $\mathrm{sign}(y) = [\mathrm{sign}(y_1), \ldots, \mathrm{sign}(y_n)]^\top$. Substituting the above choice of $g(x_p, x_t)$ in the $\dot{J}$ equation in (5) one obtains

$$\begin{aligned} \dot{J} = &-\alpha \big\| \nabla_{x_p} J(x_p, x_t) \big\|^2 \\ &- \beta \big\| \nabla_{x_p} J(x_p, x_t) \big\|_1 - \nabla_{x_p} J^\top(x_p, x_t)\dot{x}_t. \end{aligned}$$

Then, from Assumption 1 the derivative of the potential is bounded by

$$\begin{aligned} \dot{J} \leq &-\alpha \big\| \nabla_{x_p} J(x_p, x_t) \big\|^2 \\ &- \beta \big\| \nabla_{x_p} J(x_p, x_t) \big\|_1 + \gamma_t \big\| \nabla_{xp} J(x_p, x_t) \|_1, \end{aligned}$$

which, on the other hand, implies that

$$\dot{J} \leq -\alpha \big\| \nabla_{x_p} J(x_p, x_t) \big\|^2,$$

since we have $\beta \geq \gamma_t$ by choice, recovering the above result. (Note that above we explicitly used the 1-norm. However, from the equivalence of norms the same in equalities will hold for the other norms as well. Therefore, for the rest of the paper we will use only single notation for the norms.) This equation implies that as time tends to infinity we have $\dot{J} \to 0$ and $\nabla_{x_p} J(x_p, x_t) \to 0$. This, on the other hand, implies that as $t \to \infty$ we have $\|x_p - x_t\| \to c = \mathit{constant}$, since $\dot{J} \to 0$. Moreover, we have the constant $c = 0$, since the unique extremum of J in (4) (or basically $\nabla_{x_p} J(x_p, x_t) = 0$) occurs at $x_p = x_t$. Therefore, the condition in (2) will be satisfied and the pursuer will track the moving target.

The above controller requires knowledge of the position of the target together with a bound on its speed and with the help of a switching term guarantees asymptotic tracking of the target. The assumption that the position x_t of the target is known allows for exact calculation of $\nabla_{x_p} J(x_p, x_t)$ and makes it possible to implement the above method. The surface $\nabla_{x_p} J(x_p, x_t) = 0$ serves as a sliding manifold for the system and leads to convergence with the use of high enough controller gain to overcome the uncertainty in the target's speed. Intuitively, the second term in (7) allows for the detection of changes in the direction of motion of the target and helps redirect the pursuer in that direction. It is possible to relax the assumption that the position of the target is known and still track the target by knowledge of only its direction of motion (or basically the sign of $\nabla_{x_p} J(x_p, x_t)$) and a bound on its speed. Next we consider this case.

Assumption 2: *The position x_t of the moving target is not known. However, $\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big)$ is known. Moreover, the velocity of the target satisfies $\|\dot{x}_t\| \leq \gamma_t$ for some known $\gamma_t > 0$.*

For the case in which Assumption 2 holds one can just choose the pursuer dynamics to be

$$\dot{x}_p = -\beta \,\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big), \qquad (8)$$

where the constant parameter β is chosen as $\beta > \gamma_t$. For example, if $\beta = (\alpha + \gamma_t)$ for some constant $\alpha > 0$, then the derivative of the potential function in (5) satisfies

$$\dot{J} \leq -\alpha \big\| \nabla_{x_p} J(x_p, x_t) \big\|,$$

guaranteeing once more eventual capture of the target (by the same reasoning as in the above case).

The advantage of the dynamics in (7) over those in (8) is that it is possible to achieve faster convergence with a much smaller switching term. In other words, for the case in which x_t is not known the magnitude of the switching term must be larger for the same convergence speed.

One disadvantage of the above results is that the dynamics in (7) and (8) do not represent the dynamics of realistic vehicles. Therefore, the model considered in this section serves essentially as a kinematic model for pursuing of a moving target. For this reason, the procedure here mostly serves as a proof of concept for the tracking/intercepting behavior. In engineering applications with agents with particular motion dynamics one has to take into account these dynamics in order to be able to develop control algorithms to achieve the required behavior. In the next section we discuss a control algorithm based on sliding mode control theory which could be applied for agents with general fully actuated dynamics. Moreover, it can be extended to agents with different vehicle dynamics.

3. Sliding mode control for agents with vehicle dynamics

In the preceding section we showed that for a system with a target (with position x_t) and a pursuer (with position x_p), the pursuer will eventually catch the target provided that its velocity vector $\dot{x}_p$ is chosen such as to satisfy (7) or (8). In this section, we will build on these results by considering a pursuer with realistic vehicle dynamics. We will perform the analysis based on (7); however, the results hold for the case of (8) as well.

We consider a pursuer agent the dynamics of which are described by the equation

$$M(x_p)\ddot{x}_p + f_p(x_p, \dot{x}_p) = u_p, \tag{9}$$

where $x_p \in \mathbb{R}^n$ is the position of the pursuer agent, $M(x_p) \in \mathbb{R}^{n\times n}$ is the mass or inertia matrix, $f_p(x_p, \dot{x}_p) \in \mathbb{R}^n$ represents centripetal forces, Coriolis, gravitational effects and additive disturbances, and $u_p \in \mathbb{R}^n$ represents the control inputs.

For the $f_p(x_p, \dot{x}_p)$ term in the vehicle dynamics equation we assume that

$$f_p(x_p, \dot{x}_p) = f_p^k(x_p, \dot{x}_p) + f_p^u(x_p, \dot{x}_p),$$

where $f_p^k(\cdot,\cdot)$ represents the known part and $f_p^u(\cdot,\cdot)$ represents the unknown part. Also, we assume that for the range of operating conditions the unknown part is bounded. In other words, we assume that

$$\|f_p^u(x_p, \dot{x}_p)\| \le \bar{f}_p,$$

where $\bar{f}_p < \infty$ is a known constant. (Note that although here we assume that $\bar{f}_p$ is constant, the procedure will work without modification for the case of known bounded function $\bar{f}_p(t)$ as well.) Moreover, it is assumed that the mass/inertia matrix is nonsingular and lower and upper bounded by known bounds. In other words, the matrix $M(x_p)$ satisfies

$$\underline{M}\|y\|^2 \le y^\top M(x_p) y \le \bar{M}\|y\|^2,$$

where $\underline{M} > 0$ and $\bar{M} < \infty$ are known and $y \in \mathbb{R}^n$ is an arbitrary vector. Note that all these assumptions are standard and realistic.

Given the agent dynamics in (9), we would like to choose (i.e., design) the control input u_p such that as time progresses the pursuer catches the target. In other words, we would like to choose u_p such that the condition in (2) is satisfied. In order to achieve this objective, there might be several different approaches, one of which is to enforce the satisfaction of (7). In other words, if the control input is designed to enforce the velocity of the pursuer agent to satisfy (7), then in the light of the discussion in the preceding section it will guarantee the satisfaction of (2). In this section we will take exactly that approach. To this end, once more we will use the sliding mode control method. The sliding mode control technique has the property of reducing the motion (and the analysis) of the dynamics of a system to a lower dimensional space, which makes it very suitable for this application (since we want to enforce the system dynamics to obey (7), which constitutes only a part of the agent's state). We will follow a procedure similar to those in Utkin *et al.* (1991), Guldner and Utkin (1993, 1995) and Gazi (2005) for robot navigation, obstacle avoidance, and swarm aggregations.

Define the n-dimensional sliding manifold for the pursuer agent as

$$s = \dot{x}_p + \alpha\nabla_{x_p} J(x_p, x_t) + \beta\,\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big), \tag{10}$$

and note that once the agent reaches its sliding manifold (i.e., once $s=0$) we have

$$\dot{x}_p = -\alpha\nabla_{x_p} J(x_p, x_t) - \beta\,\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big),$$

which is exactly the motion equation in (7). Now, the problem is to design the control input u_p such as to enforce the occurrence of sliding mode. A sufficient condition for sliding mode to occur is given by DeCarlo *et al.* (1988)

$$s^\top \dot{s} < 0, \tag{11}$$

which also guarantees that the sliding manifold is asymptotically reached (i.e., it guarantees that the reaching conditions are satisfied). Later we will also show how to choose a controller which will actually guarantee finite time reaching of the sliding manifold. Differentiating the sliding manifold equation we obtain

$$\dot{s} = \ddot{x}_p + \frac{\mathrm{d}}{\mathrm{d}t}\big[\alpha\nabla_{x_p} J(x_p, x_t)\big] + \frac{\mathrm{d}}{\mathrm{d}t}\big[\beta\,\mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big)\big].$$

One issue to note here is that the third term on the right hand side of the above equation is unbounded at the instances at which $\nabla_{x_p} J(x_p, x_t)$ changes sign and is zero at the other time instants. However, for now, let us assume that it is bounded by a known constant $\bar{J}_s$ even at the switching instants. In other words, let us temporarily assume that

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t} \left[\beta \operatorname{sign}\left(\nabla_{x_p} J(x_p, x_t) \right) \right] \right\| \leq \bar{J}_s$$

for a known $0 < \bar{J}_s < \infty$. Moreover, we assume that the second term is also bounded, i.e.,

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t} \left[\alpha \nabla_{x_p} J(x_p, x_t) \right] \right\| \leq \bar{J}(x_p, x_t)$$

for some known $0 < \bar{J}(x_p, x_t) < \infty$. Note that this assumption is not needed in the case in which we use only the knowledge of the sign of $\nabla_{x_p} J(x_p, x_t)$ (and not the position of x_t). Moreover, it is not a strong assumption and is satisfied by many potentials. In fact, for the function in (4) it can be shown with a straightforward manipulation (see the appendix) that

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t} \left[\alpha \nabla_{x_p} J(x_p, x_t) \right] \right\| \leq \alpha \|\dot{x}_p\| + \alpha \gamma_t. \tag{12}$$

With further manipulation this bound could be rewritten in the form

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t} \left[\alpha \nabla_{x_p} J(x_p, x_t) \right] \right\| \leq \alpha^2 \|x_p - x_t\| + \alpha(\|s\| + \beta + \gamma_t). \tag{13}$$

Note that both expressions on the right hand sides of equations (12) and (13) are computable and can serve as the bound $\bar{J}(x_p, x_t)$, although (12) is preferable if $\|\dot{x}_p\|$ is available, because it is smaller. Note also that once the sliding manifold is reached $J(x_p, x_t)$ is decreasing and since the manifold is reached in a finite time (as will be shown below) we have $\bar{J}(x_p, x_t) \leq \bar{J}$ for some finite constant $\bar{J}$.

From the vehicle dynamics of the agents in (9) we have

$$\ddot{x}_p = M^{-1}(x_p) \left[u_p - f_p(x_p, \dot{x}_p) \right],$$

using which in the $\dot{s}$ equation and substituting it in (11), the condition for occurrence of sliding mode becomes

$$s^\top \Big[M^{-1}(x_p) u_p - M^{-1}(x_p) f_p(x_p, \dot{x}_p). + \frac{\mathrm{d}}{\mathrm{d}t} \left[\alpha \nabla_{x_p} J(x_p, x_t) \right] + \frac{\mathrm{d}}{\mathrm{d}t} \left[\beta \operatorname{sign}\left(\nabla_{x_p} J(x_p, x_t) \right) \right] \Big] < 0.$$

If the above boundedness assumptions hold, then one can choose the control input u_p such that $s^\top \dot{s} < 0$ is satisfied. In particular, by choosing

$$u_p = -u_0 \operatorname{sign}(s) + f_p^k(x_p, \dot{x}_p), \tag{14}$$

we obtain

$$s^\top \dot{s} < -\|s\| \left[\left(\frac{1}{\bar{M}} \right) u_0 - \left(\frac{1}{\underline{M}} \right) \bar{f}_p - \bar{J}(x_p, x_t) - \bar{J}_s \right].$$

Then, by choosing the gain u_0 of control input as

$$u_0 \geq \bar{M} \left(\frac{1}{\underline{M}} \bar{f}_p + \bar{J}(x_p, x_t) + \bar{J}_s + \epsilon \right),$$

for any $\epsilon > 0$, one can guarantee that

$$s^\top \dot{s} < -\epsilon \|s\|$$

is satisfied and that sliding mode occurs. In other words, once the sliding manifold $s = 0$ is reached, the system remains on that manifold for all time. Choose the Lyapunov function as $V = \frac{1}{2} s^\top s$ and note also that the above inequality implies that $\dot{V} \leq -\epsilon \sqrt{V}$. This, on the other hand, in light of the comparison principle (Khalil 1996), guarantees that the sliding manifold is reached in a *finite time* bounded by

$$t_{\max} = \frac{2\sqrt{V(0)}}{\epsilon} = \frac{2\|s(0)\|}{\epsilon}.$$

Then, under ideal sliding mode the behavior described in the preceding section for the "kinematic" model is recovered implying that the tracking of the target is achieved. In other words, the result can be described as follows: The sliding mode surface $s = 0$ is reached in finite time and then once it is reached the pursuer asymptotically tracks the target. This is important since it guarantees tracking of a moving target for pursuers with general vehicle dynamics with system uncertainties and additive disturbances. An important advantage of the controller is that it does not require the knowledge of the uncertainties (e.g., it does not require the knowledge of the exact mass/inertia matrix $M(x_p)$ of the pursuer robot) or the disturbances. It needs only the bounds on them. These properties constitute important advantages and are due to the robustness properties of the sliding mode control technique. Note also that in the above controller, we utilized the known part $f_p^k(x_p, \dot{x}_p)$ of the vehicle dynamics. If there are not known parts, then this portion of the controller can be set to zero.

The above results crucially depend on the assumption that the term $\mathrm{d}/\mathrm{d}t\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]$ is bounded. However, this assumption does not hold since the derivative of the signum function is unbounded on the switching instances. To overcome this problem we use an idea similar to that of the equivalent control method and sliding mode observers (Utkin 1977, DeCarlo *et al.* 1988, Drakunov 1992, Drakunov and Utkin 1995, Haskara *et al.* 1998). Recall that the equivalent control method allows the derivation of an analytical controller assuming ideal sliding mode. Moreover, it shows that the high frequency switching controller has an "average" or an "effective" value during sliding mode. Therefore, by passing the switching signal through a low pass filter it is possible to extract that value by cutting off the high frequency component. Analogously, the $\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)$ term must have an equivalent component and a high frequency component during sliding mode. Denote its equivalent component as $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}}$. Then, the bound $\bar{J}_s$ is the value which satisfies

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t}\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}} \right\| \le \bar{J}_s.$$

We would like to mention here just as a remark that at ideal sliding mode when the pursuer catches the target (i.e., $s = 0$ and $\nabla_{x_p} J(x_p, x_t) = 0$ the value of $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}}$ becomes $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}} = -\dot{x}_p = \dot{x}_t$, implying that the switching term $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]$, in a sense, estimates or approximates the unknown speed of the target.

For practical implementation, as in the sliding mode observers, the value of $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}}$ can be extracted by passing $\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)$ through an appropriate low pass filter. With this in mind define

$$\mu\dot{z} = -z + \beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right),$$

where μ is a small positive constant. In this system the high frequency switching signal $\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)$ is the input and z is the filtered output. Then, with proper choice of the parameter μ, which is the time constant of the system and the inverse of the cutoff frequency of the filter, at steady state we have

$$z \approx \left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]_{\mathrm{eq}}.$$

This equation allows us to replace $\left[\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right]$ in the sliding manifold equation in (10) with z. In other words, we redefine the sliding manifold as

$$s_{\mathrm{new}} = \dot{x}_p + \alpha\nabla_{x_p} J(x_p, x_t) + z,$$

and since z is bounded the method derived above could be implemented for this new sliding manifold. In order to be consistent with the derivation above, we assume that the bound on z is the constant $\bar{J}_s$ used above. In other words, we have

$$\|\dot{z}\| = \left\| \frac{1}{\mu}\left[-z + \beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)\right] \right\| \le \frac{2\beta}{\mu}, \qquad (15)$$

which holds since $\|z\| \le \beta$. Then, the controller in (14) with s replaced with s_{new}, i.e.,

$$u_p = -u_0\,\mathrm{sign}(s_{\mathrm{new}}) + f_p^k(x_p, \dot{x}_p),$$

with gain u_0 chosen as before and $\bar{J}_s \triangleq (2\beta/\mu)$, guarantees the occurrence of sliding mode at the new (redefined) manifold s_{new} in a finite time.

The idea of utilizing z instead of the switching term $\beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right)$ in the sliding manifold equation is a key idea of this article, which makes the algorithm implementable. However, it comes also at a price, since s_{new} is not exactly equal to s. In other words, at the new sliding manifold $s_{\mathrm{new}} = 0$ the motion of the pursuer agent becomes

$$\begin{aligned}\dot{x}_p &= -\alpha\nabla_{x_p} J(x_p, x_t) - z \\ &= -\alpha\nabla_{x_p} J(x_p, x_t) - \beta\,\mathrm{sign}\left(\nabla_{x_p} J(x_p, x_t)\right) + \mu\dot{z}.\end{aligned}$$

For this new $\dot{x}_p$ one can show that in the worst case the time derivative of the potential function $J(x_p, x_t)$ is bounded by

$$\dot{J} \le -\alpha\left\|\nabla_{x_p} J(x_p, x_t)\right\|\left[\left\|\nabla_{x_p} J(x_p, x_t)\right\| - \frac{\beta + \gamma_t}{\alpha}\right]$$

which guarantees that $J(x_p, x_t)$ decreases for

$$\left\|\nabla_{x_p} J(x_p, x_t)\right\| > \frac{\beta + \gamma_t}{\alpha}$$

which for the given potential in (4) means

$$\left\|x_p - x_t\right\| > \frac{\beta + \gamma_t}{\alpha}. \qquad (16)$$

In other words, with the introduction of the filter one can guarantee only bounded tracking of the target. Still, however, by choosing the parameter α high enough the tracking error can be made as small as desired. In particular, for a desired bound of δ on the

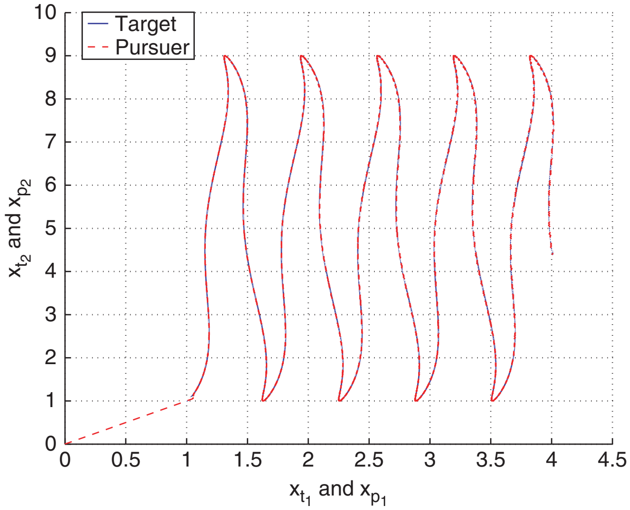

Figure 1. The trajectories of the target and the pursuer (the kinematic model case).

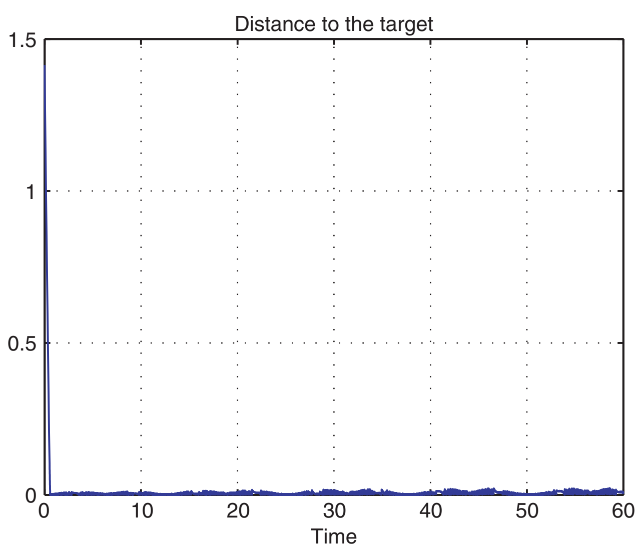

Figure 2. The distance between the target and the pursuer (the kinematic model case).

tracking error, by choosing

$$\alpha = \frac{2\beta}{\delta}$$

one can guarantee that as $t \to \infty$ we have

$$\lim_{t\to\infty} \|x_p - x_t\| \le \delta.$$

This completes the development of the sliding mode controller. Note that the controller consists of two stages (as is the case of all sliding mode controllers). The first stage is the definition of an appropriate sliding manifold. This was performed in the preceding section. In other words, while discussing the "kinematic" model for the pursuer we also defined a sliding surface for the dynamic model of this section. The second stage of the sliding mode control design is to enforce occurrence of sliding mode on the designed surface and this was discussed in this section. As a difference from usual sliding surfaces, the sliding manifold considered here contains a switching term with unbounded first derivative. This difficulty was overcome by using ideas from the equivalent control method and the sliding mode observers. In particular, for practical implementation by redefining the manifold and replacing the switching term with its smooth steady state equivalent, obtained by and appropriate lowpass filet, one can achieve bounded tracking with user defined arbitrary accuracy. In the next section we will provide a few simulation examples illustrating the behavior of the system.

4. Simulation examples

In this section some numerical simulation examples will be presented in order to illustrate the effectiveness of the sliding mode controller for intercepting moving targets. For ease of plotting we use only $n = 2$; however, qualitatively the results will be the same for higher dimensions. First, we will provide a few simulations for the "kinematic" model and after that we will consider agents (robots) with point mass dynamics with unknown mass and unknown but bounded additive disturbances. In all the simulations below we used $\alpha = 0.01$ and $\beta = 2.0$ as the controller parameters.

Figure 1 shows a simulation for the case with the "kinematic" model. Initially the target is located at the position $[1, 1]$ in the plane, whereas the pursuer is located at the origin. The target tries to escape following a sinusoidal type of trajectory, according to the dynamics

$$\dot{x}_{t_1} = 0.05 + 0.1\sin(2t)$$
$$\dot{x}_{t_2} = 1.9\sin(0.5t).$$

As one can see that the pursuer catches up with the target in a short period of time and follows it after that. Similar results are obtained for other trajectories of the target such as trajectory generated with a random velocity. Figure 2 shows the distance between the target and the pursuer. We believe that the fact that the distance between the two does not vanish is due to the chattering effects (which arise from the numerical errors in Matlab in this case due to the stiffness of the function – here we used the signum function in the simulation). Note that the chattering

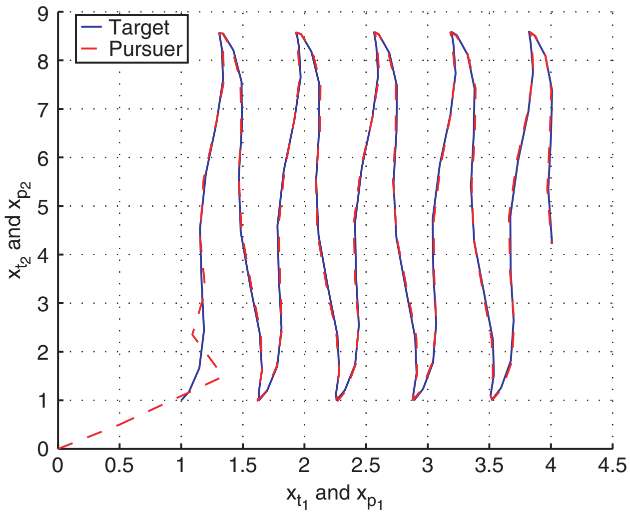

Figure 3. The trajectories of the target and the pursuer for the case with $\mu = 0.5$ (the dynamic model case).

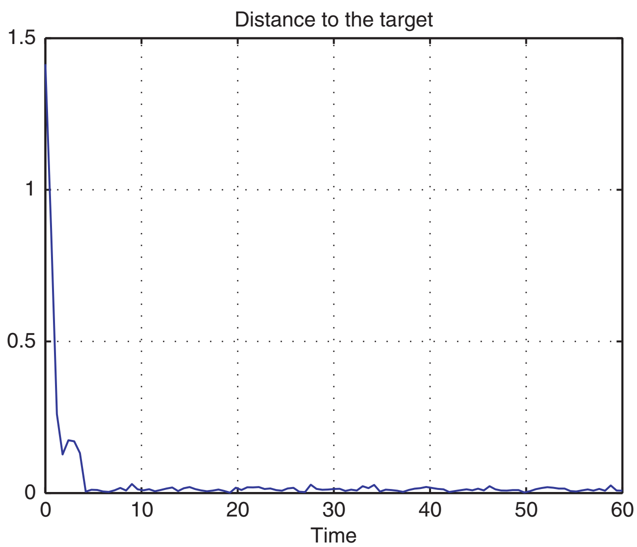

Figure 4. The distance between the target and the pursuer for the case with $\mu = 0.5$ (the dynamic model case).

effect can also be seen from figure 1, where the pursuer trajectory crosses back and forth the trajectory of the target. These are consistent with the theoretical expectation discussed in the preceding sections. Note that there are ways to reduce or eliminate chattering (Bondarev *et al.* 1985). However, these are outside the scope of this article.

Next, consider agents (robots) with point mass dynamics with unknown mass and unknown but bounded additive disturbances. In other words, we consider the model

$$M_p \ddot{x}_p + f_p(x_p, \dot{x}_p) = u_p,$$

where $\underline{M} \leq M_p \leq \bar{M}$ is the unknown mass and $f_p(x_p, \dot{x}_p) = \sin(0.2t)$ is the uncertainty in the system. Without loss of generality we assume unity mass $M_p = 1$ for the agent. In the simulations below as controller parameters we choose $\underline{M} = 0.5$ and $\bar{M} = 1.5$, $\bar{f}_p = 1$ and $\epsilon = 1$. Moreover, we also replaced the $\text{sign}(s_{\text{new}})$ term in the controller with the term $\tanh(\eta s_{\text{new}})$ (which also contributes to the tracking error in the simulation), with $\eta = 10$. This smooths the control action and is often used instead of the discontinuous signum function in sliding mode control applications. The parameter η is a smoothness parameter which determines the slope of the curve around zero.

Figure 3 shows the trajectories for the target and the pursuer for the case with the above dynamic model with the same initial positions as the previous case and pursuer with zero initial velocity. Here, we have $\bar{J}$ computed according to (13), and $\bar{J}_s = 8$ (found by evaluating equation (15)) for the bounds on $\|(\mathrm{d}/\mathrm{d}t)[\alpha \nabla_{x_p} J(x_p, x_t)]\|$ and $\dot{z}$, respectively. As one can easily see from the figure the trajectories for this case are very similar to those in figure 1 which were obtained for the kinematic model. Figure 4 shows the distance between the target and the pursuer. It is observed that for this case the error is larger and approaches zero slower compared to the earlier case. This is due to the fact that the lowpass filter used was not adequate and is unable to extract the actual "average" value of its input. For this case we used a filter with time constant $\mu = 0.5$. Note the interesting fact that the actual tracking performance of the method is much better than predicted by the theory. Indeed, if we evaluate the expected bound (16) for the steady-state distance between pursuer and target, we obtain 400, while the actual bound at steady state in figure 4 is about 0.06! Clearly, the theoretical bound is so large because, in choosing $\alpha = 0.01$, a relatively small number, we attempt to reduce the control energy used. This bound does not depend on the choice for μ; therefore, we can expect a performance degradation with increasing values of μ, but no worse than the bound given in (16). Consequently, if we choose, for instance, $\alpha = 10$, then the tracking performance improves significantly, regardless of μ, with a theoretical tracking error of at most 0.4 and actual/practical tracking error—not shown here—of at most 0.0015 in steady-state.

Figures 5 and 6 show the results for the case in which the filter parameter was decreased to $\mu = 0.1$. Here, we set $\bar{J}_s = 40$ from (15). One can easily see that for this case the error is much smaller compared to the earlier case – it is about 0.01 at steady state, which is six times less than the previous case. This is because now the filter works properly and therefore we have $s_{\text{new}} \approx s$ and the result for the kinematic case is recovered.

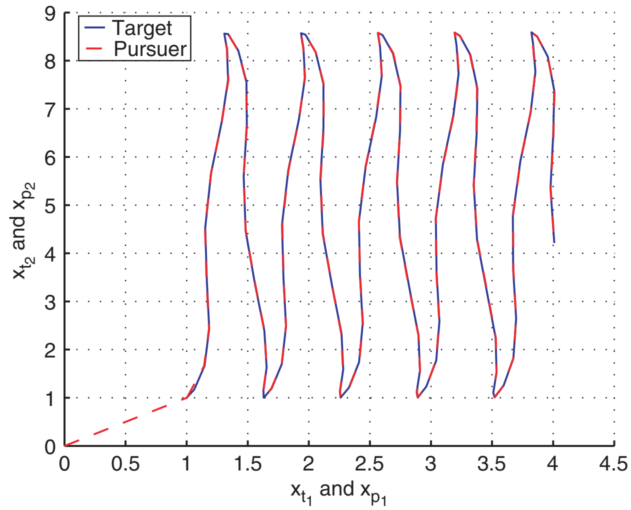

Figure 5. The trajectories of the the target and the pursuer for the case with $\mu = 0.1$ (the dynamic model case).

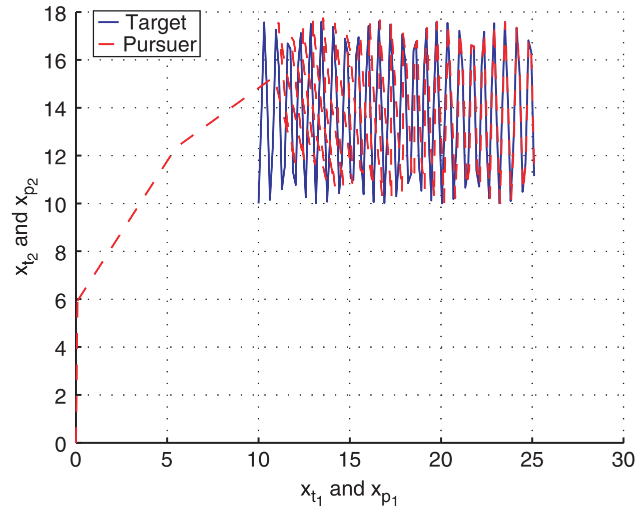

Figure 7. The trajectories of the target and the pursuer for the case with an obstacle at [5, 5] (the dynamic model case).

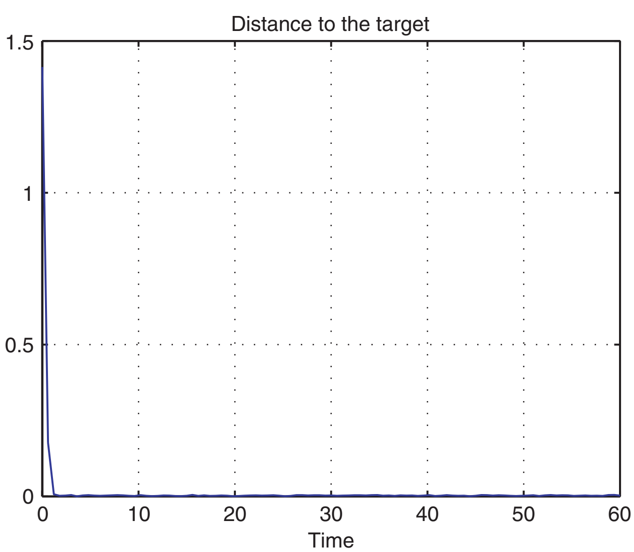

Figure 6. The distance between the target and the pursuer for the case with $\mu = 0.1$ (the dynamic model case).

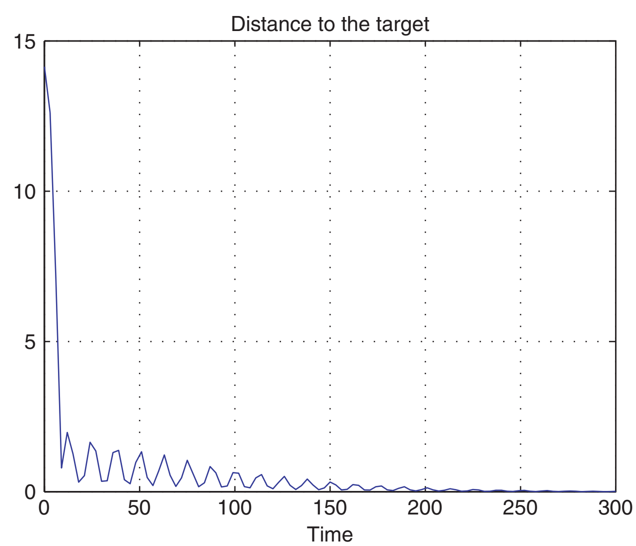

Figure 8. The distance between the target and the pursuer for the case with an obstacle at [5, 5] (the dynamic model case).

Finally, in figures 7 and 8 we provide a simulation result for a case not discussed in the preceding sections. There, we included an obstacle at the position [5, 5] and modeled it as a Gaussian potential centered at that point with magnitude 50 and spread 2. Moreover, we included this potential in the potential J for the inter-individual distance between the target and the pursuer. It was assumed that the target this time is located at the position [10, 10]. Here, we set $\mu = 0.1$ as before and keep the same bounds $\bar{J}$ and $\bar{J}_s$. As one can easily see from the figure, the pursuer avoids the obstacle and is in the end able to catch the target (which moves in a manner similar to before). This time it takes longer for the pursuer to intercept the target (we run the simulation for 300 seconds, whereas in the previous cases we run it only for 60 seconds), which is expected. This shows the potential of the algorithm: it might be possible to use it for capturing/intercepting moving targets in a structured environment. However, this still needs to be carefully considered.

5. Concluding remarks

In this article, we presented a procedure based on sliding mode control theory which can be used to intercept/capture a moving target. One of the advantages of the method is that it is robust with respect to disturbances and system uncertainties. The algorithm is also promising from the perspective that it might be possible to use it for intercept/capture of targets moving in a structured environment. Moreover, it may be possible to use a similar method for capturing/enclosing a moving target by making group formations by multiple pursuers. Here we did not impose velocity constraints on the pursuer. The performance of the algorithm under such constraints can also be investigated in future work.

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Appendix. Derivation of $\bar{J}(x_p, x_t)$

Given the potential function in (4) its gradient at x_p is given by

$$\nabla_{x_p} J(x_p, x_t) = (x_p - x_t).$$

Then, the bound on its derivative with respect to time can be calculated as

$$\begin{aligned} \left\| \frac{\mathrm{d}}{\mathrm{d}t} \alpha \nabla_{x_p} J(x_p, x_t) \right\| &= \left\| \frac{\mathrm{d}}{\mathrm{d}t} \alpha (x_p - x_t) \right\| \\ &= \| \alpha (\dot{x}_p - \dot{x}_t) \| \\ &\leq \alpha \| \dot{x}_p \| + \alpha \gamma_t. \end{aligned} \tag{17}$$

Note that this bound is computable and can be used as $\bar{J}(x_p, x_t)$. Another possible bound could be obtained by using the equality

$$\dot{x}_p = s - \alpha \nabla_{x_p} J(x_p, x_t) - \beta \, \mathrm{sign}\big(\nabla_{x_p} J(x_p, x_t)\big),$$

in the above equation and manipulating further. Basically, from the above equality we have

$$\| \dot{x}_p \| \leq \| s \| + \alpha \| \nabla_{x_p} J(x_p, x_t) \| + \beta$$

substituting which in (17) one obtains

$$\left\| \frac{\mathrm{d}}{\mathrm{d}t} \big[\alpha \nabla_{x_p} J(x_p, x_t) \big] \right\| \leq \alpha^2 \| x_p - x_t \| + \alpha (\| s \| + \beta + \gamma_t),$$

which is also a computable bound.

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