

Swarm Tracking Using Artificial Potentials and Sliding Mode Control

Jingyi Yao

Diebold, Inc.

e-mail: yaoj@diebold.com

Raúl Ordóñez

Associate Professor

Department of Electrical and Computer Engineering,

University of Dayton,

300 College Park,

Dayton, OH 45469-0232

e-mail: ordonez@ieee.org

Veysel Gazi

Department of Electrical and Electronics Engineering,

TOBB University of Economics and Technology,

Söğütözü Caddesi No. 43,

06560 Ankara, Turkey

e-mail: vgazi@etu.edu.tr

In this paper, we present a stable and decentralized control strategy for multiagent systems (swarms) to capture a moving target in a specific formation. The coordination framework uses artificial potentials to take care of both tracking and formation tasks. First, a basic controller is designed based on a kinematic model. After that, sliding mode control technique is used to force the agents with general vehicle dynamics to obey the required motion. Finally, specific potential functions are discussed and corresponding simulation results are given. [DOI: 10.1115/1.2764511]

1 Introduction

In recent years, it has become popular to study the biological world and apply the derived principles to the design of engineering systems. The topic of distributed coordination and control of multiple autonomous agents has gained lots of attention [1–3]. Cooperative agents can often be used to perform tasks that are too difficult for a single one to perform. Instead of the traditional trajectory tracking problem, people began to study coordinated tracking [4,5]. Tasks can be performed more efficiently by controlling the group to move in formation. Possible applications could range from autonomous robot assembly to unmanned aerial vehicles (UAV) scout and counterwork.

The coordinated tracking problem is to find a coordinated control scheme for a group of agents to make them achieve and maintain some given geometrical formation; at the same time, the agents viewed as a group have to track a target or a trajectory (e.g., which may be required in order to execute a given task). Thus, there is a trade-off between maintaining formation and arriving at the final goal. Possible approaches for formation control include leader-following and the virtual structure approaches [6,7]. In both approaches, one agent, which could be real or virtual, is designated as leader to perform the tracking task without considering the followers, and the remaining agents only have to stay from the leader within a desired offset, without considering the target. However, these approaches are centralized and there-

fore not robust; if the leader fails, the task would also fail. In that case, decentralized formation control is preferred as discussed in Ref. [8], where feedback control laws are used to keep the formation during tracking process.

In this paper, we consider a control strategy based on artificial potential functions and sliding mode control. Artificial potential functions have been widely used for robot navigation and control including multiagent coordination [6,9,10]. The potential function is created to encode the interaction rule for the group. In order to perform the task in a decentralized way, in this paper, we consider a potential function composed of two parts. The interconnection part makes the agent constrained by its neighbor to maintain a group structure, while pursuer-target potential function is introduced to direct the group behavior, which is to catch up with the target. The specific form of potential function is defined according to the desired geometric formation.

We show that by appropriate choice of the potential function, one can always guarantee that eventually the target will be surrounded or “enclosed” by the tracking agents. One advantage provided by being able to surround and track a target with an arbitrary formation is that, in both military and civilian applications, vehicles typically have sensors that can only work—or work best—when pointed at the target from a certain angle.

The sliding mode control technique has the important properties of suppressing disturbances and model uncertainties. It becomes attractive for two main advantages: (i) the dynamic behavior of the system may be tailored by the particular choice of switching function, and (ii) the closed-loop response becomes totally insensitive to a particular class of uncertainties. This idea has already been successfully used to implement engineering aggregating swarm [11] and tracking moving targets [12]. In this paper we extend the work to the case of multiple agents tracking or capturing/enclosing a moving target (possibly) in a formation. Initial version of this paper can be found in Ref. [13].

The paper is organized as follows: In Sec. 2, we define a general potential function and develop the control algorithm for the “kinematic” model. In Sec. 3, we consider a general fully actuated dynamic model of the agents and derive a new controller based on the sliding mode control method. In Sec. 4, specific potential functions are applied to the problem and results are presented and discussed. Finally, conclusions are made in Sec. 5.

2 Basic “Kinematic Model”

Consider a multiagent system (i.e., a swarm) consisting of N individuals in an n -dimensional Euclidean space. To begin with, we model the individuals as points and ignore their dimensions. Moreover, we assume synchronous motion and no time delays. Let $\mathbf{x}_i, \mathbf{x}_t \in \mathbb{R}^n$ denote the position vector of individual i and the target, respectively. Also, temporarily assume that the motion dynamics for the agents are given by

$$\dot{\mathbf{x}}_i = \mathbf{u}_i \quad (1)$$

which we call the kinematic model. Later, we will show how the derivations in this section can be extended to the case in which the agents have fully actuated dynamics. Our objective is to make the entire group aggregate around the target and move together with it possibly in a specific formation regardless of the target’s movement. This is a simple case of the coordinated tracking problem.

We can design a controller in a simpler form if $\dot{\mathbf{x}}_t$ is available [12]. However, assuming that $\dot{\mathbf{x}}_t$ is known is a strong assumption since the current velocity of the target is unknown in most circumstances. It is more realistic to assume that $\|\dot{\mathbf{x}}_t\| \leq \gamma_t$ for some known $\gamma_t > 0$ since any realistic agent has a bounded velocity. It is assumed that the agents are able to move faster than the target and there is no explicit upper bound on the velocity of the agents. We also assume that the relative position of the target \mathbf{x}_t is known and each agent knows the exact relative positions of all the other individuals.

Contributed by the Dynamic Systems, Measurement, and Control Division of ASME for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received March 31, 2006; final manuscript received February 12, 2007. Review conducted by Prabhakar R. Pagilla.

With the assumptions above, we choose the control laws such that each agent is moving based on the equation

$$\dot{\mathbf{x}}_i = \mathbf{u}_i = -\alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) - \beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)] \quad (2)$$

for all $i=1, \dots, N$ where $\alpha > 0$ and $\beta \geq \gamma_i$ are positive constants, $\operatorname{sgn}(\cdot)$ is the signum function operated elementwise for a vector $\mathbf{y} \in \mathbb{R}^n$, and $J: \mathbb{R}^{n \times N} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a potential function (to be defined below). It is specified by the multiagent system designer based on the desired structure and/or behavior of the swarm. Such potential functions are being used for swarm aggregations, formation control, and multiagent coordination and so on [11,14]. Note also that in the above equation, we implicitly defined $\mathbf{x}^T = [\mathbf{x}_1^T, \dots, \mathbf{x}_N^T] \in \mathbb{R}^{nN}$.

In order to satisfy both the tracking and formation control specifications, we consider potential functions $J(\mathbf{x}, \mathbf{x}_i)$, which are composed of two parts: the interagent interactions (or formation control) part and the agent-target interaction (or tracking) part. In particular, we consider potential functions of the form

$$J(\mathbf{x}, \mathbf{x}_i) = K_T \sum_{i=1}^N J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|) + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) \quad (3)$$

where $J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|)$ is the potential between agent and target, while $J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|)$ is the potential between agents in the group. With such a form, each agent takes care of the tracking task by itself and keeps certain distances between itself and its neighbors. For the tracking part, it is required that $J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|)$ has a unique minimum at a particular distance from the target so that the individuals catch up with the target and encircle/enclose it. As for the formation part, $J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|)$ is required also to have a unique minimum at the desired distance between the agent based on the formation expected to be achieved. Note that $J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|)$ can be different for different pairs. (The same could be the case for $J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|)$ as well although we used the same $J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|)$ for all i in this paper.) The coefficients K_T and K_F weigh the relative importance of tracking versus formation keeping.

When the task is achieved, which means each agent reaches the desired distances from both target and other agents, J equals to a desired minimum. The potential function defined in Eq. (3) is based on relative positions instead of absolute positions. Hence, in terms of the states, J has a family of minima for the same formation, for example, due to rotations and translations.

Assume $J_{it}(\|\mathbf{x}_i - \mathbf{x}_t\|)$ satisfies the following.

- (a) There exists corresponding function $h^{it}: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_{\mathbf{y}} J_{it}(\|\mathbf{y}\|) = \mathbf{y} h^{it}(\|\mathbf{y}\|) \quad (4)$$

- (b) There exist unique distances δ_{it} at which we have $h^{it}(\|\mathbf{y}\|) = 0$.

Assume $J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|)$ satisfies the following.

- (a) The potentials $J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|)$ are symmetric and satisfy

$$\nabla_{\mathbf{x}_i} J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) = -\nabla_{\mathbf{x}_j} J_{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) \quad (5)$$

- (b) There exists corresponding function $g^{ij}: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_{\mathbf{y}} J_{ij}(\|\mathbf{y}\|) = \mathbf{y} g^{ij}(\|\mathbf{y}\|) \quad (6)$$

- (c) There exist unique distances δ_{ij} at which we have $g^{ij}(\|\mathbf{y}\|) = 0$.

Then,

$$\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) = K_T (\mathbf{x}_i - \mathbf{x}_t) h^{it}(\|\mathbf{x}_i - \mathbf{x}_t\|) + K_F \sum_{j=1, j \neq i}^N (\mathbf{x}_i - \mathbf{x}_j) g^{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) \quad (7)$$

$$\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) = -K_T \sum_{i=1}^N (\mathbf{x}_i - \mathbf{x}_t) h^{it}(\|\mathbf{x}_i - \mathbf{x}_t\|) \quad (8)$$

By observing the equalities in Eqs. (7) and (8), the equality in Eq. (8) can be rewritten as

$$\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) = -\sum_{i=1}^N \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) + K_F \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\mathbf{x}_i - \mathbf{x}_j) g^{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) \quad (9)$$

Moreover, since we have

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N (\mathbf{x}_i - \mathbf{x}_j) g^{ij}(\|\mathbf{x}_i - \mathbf{x}_j\|) = 0 \quad (10)$$

which follows from Eq. (5), we obtain

$$\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) = -\sum_{i=1}^N \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) \quad (11)$$

The time derivative of function J is given by

$$\dot{J} = \sum_{i=1}^N [\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)]^T \dot{\mathbf{x}}_i + [\nabla_{\mathbf{x}_t} J(\mathbf{x}, \mathbf{x}_t)]^T \dot{\mathbf{x}}_t \quad (12)$$

Substituting the agent dynamics in Eq. (2) and the condition in Eq. (11) in the \dot{J} equation, one obtains

$$\begin{aligned} \dot{J} &= -\sum_{i=1}^N [\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)]^T \{ \alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) \\ &\quad + \beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)] \} - \sum_{i=1}^N [\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)]^T \dot{\mathbf{x}}_i \\ &\leq -\alpha \sum_{i=1}^N \|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\|^2 - \beta \sum_{i=1}^N \|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\| + \gamma_i \sum_{i=1}^N \|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\| \end{aligned} \quad (13)$$

Since we have $\beta \geq \gamma_i$, the time derivative of J is bounded by

$$\dot{J} \leq -\alpha \sum_{i=1}^N \|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\|^2 \quad (14)$$

This equation implies that as time tends to infinity, we have $\dot{J} \rightarrow 0$, which indicates J converges to a constant value implying that the system converges to a configuration corresponding to a minimum of J .

This is because from LaSalle Yoshizawa theorem, Eq. (14) implies that $\lim_{t \rightarrow \infty} \sum_{i=1}^N \|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\|^2 = 0$, implying $\|\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i)\| \rightarrow 0$ for all i . Also, from Eq. (11), we see that we also have $\|\nabla_{\mathbf{x}_t} J(\mathbf{x}, \mathbf{x}_t)\| \rightarrow 0$. In other words, as $t \rightarrow \infty$, we have $(\mathbf{x}, \mathbf{x}_t) \rightarrow \Omega \subset \{(\mathbf{x}, \mathbf{x}_t) | \dot{J} = 0\}$, where

$$\Omega = \{(\mathbf{x}, \mathbf{x}_t) | \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_i) = 0, \nabla_{\mathbf{x}_t} J(\mathbf{x}, \mathbf{x}_t) = 0, i = 1, \dots, N\}$$

If the potential function we choose has one unique minimum, we will achieve our goals. Unfortunately, most potential functions may have multimimima. If the distance between any pair of objects including agents and target ($\|\mathbf{x}_i - \mathbf{x}_t\|$ and $\|\mathbf{x}_i - \mathbf{x}_j\|$) is the desired one (δ_{it} and δ_{ij}), the potential function J reaches its global minimum, which corresponds to our tracking and formation objectives. However, there are other feasible configurations different from the desired one, at which the potential function J achieves a local minimum. Therefore, in general, unless the initial configuration of the agents is "close enough" to the global minimum, it might be the case that J converges to such a local minimum resulting in a different configuration.

Then, from Eq. (8), we have $(\mathbf{x}, \dot{\mathbf{x}}_i)$ in Ω satisfying

$$-K_T \sum_{i=1}^N (\mathbf{x}_i - \mathbf{x}_t) h''(\|\mathbf{x}_i - \mathbf{x}_t\|) = 0$$

Rearranging this equation, we obtain

$$\sum_{i=1}^N \mathbf{x}_i h''(\|\mathbf{x}_i - \mathbf{x}_t\|) = \mathbf{x}_t \sum_{i=1}^N h''(\|\mathbf{x}_i - \mathbf{x}_t\|) \quad (15)$$

which is guaranteed to be achieved as $t \rightarrow \infty$. This is an important observation because it provides a relation of the position of the target to the position of the agents at equilibrium and allows the designer to appropriately choose the $h''(\|\mathbf{x}_i - \mathbf{x}_t\|)$ (the tracking part of the potential function).

First of all, note that at the desired formation, since $\|\mathbf{x}_i - \mathbf{x}_t\| = \delta_{it}$ for all i , we have $h''(\|\mathbf{x}_i - \mathbf{x}_t\|) = 0$ for all i and Eq. (15) is satisfied. In this case, agents catch up with the target and compose the expected formation with it.

Second issue to note is that if $h''(\|\mathbf{x}_i - \mathbf{x}_t\|)$ are chosen such that $\sum_{i=1}^N h''(\|\mathbf{x}_i - \mathbf{x}_t\|) = 0$ is feasible (excluding the case at the desired formation), then the position of the target \mathbf{x}_t cannot be specified (meaning that it could be anywhere in the state space). To avoid this situation, one can choose $h''(\|\mathbf{y}\|) > 0$ for all \mathbf{y} except $\|\mathbf{y}\| = \delta_{it}$. Then, assuming that $\sum_{i=1}^N h''(\|\mathbf{x}_i - \mathbf{x}_t\|) \neq 0$, we obtain

$$\mathbf{x}_t = \frac{\sum_{i=1}^N \mathbf{x}_i h''(\|\mathbf{x}_i - \mathbf{x}_t\|)}{\sum_{i=1}^N h''(\|\mathbf{x}_i - \mathbf{x}_t\|)}$$

Defining

$$\eta_i = \frac{h''(\|\mathbf{x}_i - \mathbf{x}_t\|)}{\sum_{i=1}^N h''(\|\mathbf{x}_i - \mathbf{x}_t\|)} \quad i = 1, \dots, N$$

we obtain

$$\mathbf{x}_t = \sum_{i=1}^N \eta_i \mathbf{x}_i$$

With the choice of $h''(\|\mathbf{y}\|) \geq 0$ for all \mathbf{y} , we see that $0 \leq \eta_i \leq 1$ for all i and $\sum_{i=1}^N \eta_i = 1$ implying that as $t \rightarrow \infty$, we will have $\mathbf{x}_t \rightarrow \text{conv}\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$, where $\text{conv}\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ is the convex hull of the positions of the agents. In other words, by choosing $h''(\|\mathbf{y}\|)$ as above, one can guarantee that as $t \rightarrow \infty$, the agents will "surround" or enclose the target, which is an important result.

Although we have not explicitly addressed environmental effects and collision avoidance with obstacles in the environment here, such issues can easily be incorporated within the framework by adding a potential function based environmental model or potential function based collision avoidance terms to the desired agent motions.

Despite the beauty of the above results, there is one shortcoming, which is that the model in Eq. (1) does not represent the dynamics of realistic agents. The results derived are still of interest since they serve as proof of concept for the behavior considered. Although they do not specify how that desired behavior could be achieved in engineering applications with given agent dynamics, they can serve as guidelines for designing such applications. In the next section, we discuss a control algorithm based on sliding mode control theory, which could be applied for agents with general fully actuated dynamics (such as omnidirectional robots).

3 Sliding Mode Control for Agents With Vehicle Dynamics

In this section, we consider that all the agents in the system have the same dynamics, which could be described by the equation

$$\mathbf{M}(\mathbf{x}_i) \ddot{\mathbf{x}}_i + \mathbf{f}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i) = \mathbf{u}_i \quad (16)$$

where $\mathbf{x}_i \in \mathbb{R}^n$ is the position of the agent, $\mathbf{M}(\mathbf{x}_i) \in \mathbb{R}^{n \times n}$ is the mass or inertia matrix, and $\mathbf{u}_i \in \mathbb{R}^n$ represents the control inputs. The function $\mathbf{f}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i) \in \mathbb{R}^n$ represents disturbances and other effects and we assume that

$$\mathbf{f}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i) = \mathbf{f}_i^k(\mathbf{x}_i, \dot{\mathbf{x}}_i) + \mathbf{f}_i^u(\mathbf{x}_i, \dot{\mathbf{x}}_i)$$

where $\mathbf{f}_i^k(\cdot, \cdot)$ represents the known part and $\mathbf{f}_i^u(\cdot, \cdot)$ represents the unknown part. For the unknown part, we assume that $\|\mathbf{f}_i^u(\mathbf{x}_i, \dot{\mathbf{x}}_i)\| \leq \bar{f}_i$, where $\bar{f}_i < \infty$ is a known constant. Moreover, it is assumed that for all \mathbf{x}_i , the matrix $\mathbf{M}(\mathbf{x}_i)$ satisfies

$$\underline{M} \|\mathbf{y}\|^2 \leq \mathbf{y}^T \mathbf{M}(\mathbf{x}_i) \mathbf{y} \leq \bar{M} \|\mathbf{y}\|^2$$

where \underline{M} and \bar{M} are known and $\mathbf{y} \in \mathbb{R}^n$ is arbitrary.

Given the dynamics above, we would like to choose the control input \mathbf{u}_i to enforce the velocity of the agent to satisfy Eq. (2). Then, the discussion in the preceding section guarantees that the group catches up with the target, or at least encloses it, and forms the desired shape. Here, we will use the sliding mode control theory, following a procedure similar to that in Ref. [12].

Define the n -dimensional sliding manifold for agent i as

$$\mathbf{s}_i = \dot{\mathbf{x}}_i + \alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t) + \beta \text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)] \quad (17)$$

Now, let us design the control input \mathbf{u}_i to enforce the occurrence of sliding mode. If $\mathbf{s}_i^T \dot{\mathbf{s}}_i < 0$, the sliding manifold is asymptotically reached. Differentiating the sliding manifold equation, we obtain

$$\dot{\mathbf{s}}_i = \ddot{\mathbf{x}}_i + \frac{d}{dt} [\alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)] + \frac{d}{dt} \{\beta \text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\} \quad (18)$$

Let us assume for now that $\|d/dt\{\beta \text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\}\| \leq \bar{J}_s$ and $\|d/dt[\alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\| \leq \bar{J}$, where \bar{J}_s and \bar{J} are known positive constants. Since the potential function is to be chosen by the designer, he or she should make sure that $\|d/dt[\alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\| \leq \bar{J}$ is satisfied for some \bar{J} . Moreover, notice that $\|d/dt\{\beta \text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\}\|$ is unbounded at the instances at which $\text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]$ is discontinuous. This problem will be solved by using a low-pass filter and will be discussed below.

By choosing

$$\mathbf{u}_i = -u_0 \text{sgn}(\mathbf{s}_i) + \mathbf{f}_i^k(\mathbf{x}_i, \dot{\mathbf{x}}_i) \quad (19)$$

and

$$u_0 > \bar{M} \left(\frac{1}{\bar{M}} \bar{f}_i + \bar{J} + \bar{J}_s + \epsilon \right) \quad (20)$$

we can guarantee that $\mathbf{s}_i^T \dot{\mathbf{s}}_i < -\epsilon \|\mathbf{s}_i\|$ for any $\epsilon > 0$, which implies that the manifold is reached in finite time.

Once the sliding manifold is reached, the system remains on that manifold for all time. Then, the results discussed in the preceding section are recovered despite the model uncertainties in Eq. (16). This result is achieved, thanks to the robustness properties of the sliding mode control method.

In order to derive the above result, we assumed that $\|d/dt\{\beta \text{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\}\| \leq \bar{J}_s$, which is not the case. This problem could be solved by passing the switching signal through a low pass filter [12]. Define

$$\mu \dot{\mathbf{z}} = -\mathbf{z} + \beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)] \quad (21)$$

where μ is a small positive constant. With proper choice of the parameter μ and using an approach similar to one of the sliding mode observers based on the equivalent control method [15], we have

$$\mathbf{z} \approx \{\beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\}_{\text{eq}} \quad (22)$$

where we used the subscript eq to denote the *equivalent* (effective or average) value of the discontinuous signal. Therefore, although $\beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]$ is not differentiable, its approximation \mathbf{z} is differentiable and can be used in the definition of the sliding manifold.

Moreover, we have

$$\left\| \frac{d}{dt} \{\beta \operatorname{sgn}[\nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t)]\}_{\text{eq}} \right\| = \|\dot{\mathbf{z}}\| \leq \frac{2\beta}{\mu} \triangleq \bar{J}_s \quad (23)$$

Therefore, the sliding manifold can be redefined as

$$\mathbf{s}_{i,\text{new}} = \dot{\mathbf{x}}_i + \alpha \nabla_{\mathbf{x}_i} J(\mathbf{x}, \mathbf{x}_t) + \mathbf{z} \quad (24)$$

Utilizing \mathbf{z} in the sliding manifold makes the algorithm implementable. One issue to notice is that μ has to be chosen properly so that the low-pass filter is able to extract the actual "average" or equivalent value of its input.

The agent vehicle dynamics here are assumed to be fully actuated. The method could be extended to systems composed of agents with nonholonomic constraints in the vehicle dynamics as well. However, extending the procedure to such systems needs further considerations and research and is out of the scope of the current paper. In addition, although the method is decentralized in the sense that each agent has a local controller and there is no leader in the swarm, it is not scalable well since it is based on the assumption that all agents know the positions of (or basically can communicate with) all the other agents in the group. Communication bandwidth limitations and communication delays could be explored in the future.

4 Potential Function Cases Discussion

As mentioned above, the algorithm may have different performance based on the chosen potential function $J(\mathbf{x}, \mathbf{x}_t)$.

In this section, we illustrate how the choice of potential function influences the application by presenting several examples. We set $n=2$ here, but the results should work for higher dimensions as well.

Consider the group consisting of agents with the same dynamics

$$\mathbf{M}(\mathbf{x}_i) \ddot{\mathbf{x}}_i + \mathbf{f}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i) = \mathbf{u}_i \quad (25)$$

and unity mass $\mathbf{M}_i=1$. Let $\mathbf{f}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i) = [\sin(0.2t), \sin(0.2t)]$ be the unknown uncertainty in the system. Assume that we know the bounds $\underline{M}=0.5$, $\bar{M}=1.5$, and $\bar{\mathbf{f}}_i=1$.

Assume the target dynamics as

$$\dot{x}_1 = 0.25$$

$$\dot{x}_2 = \sin(0.25t)$$

We set the initial conditions of agents randomly within a circle with $R=5$ near the origin and set the target initially at $[5, 5]$, which is located out of the agents.

Let $N=4$, our task is to make the four agents form a diamond with the target in the midpoint. Let $\delta_{ij}=2$ be the distance between agents, and the distance between the target and agents $\delta_{it}=1$ or $\delta_{it}=\sqrt{3}$.

Case 1. For this case, we chose the potential function as [16]

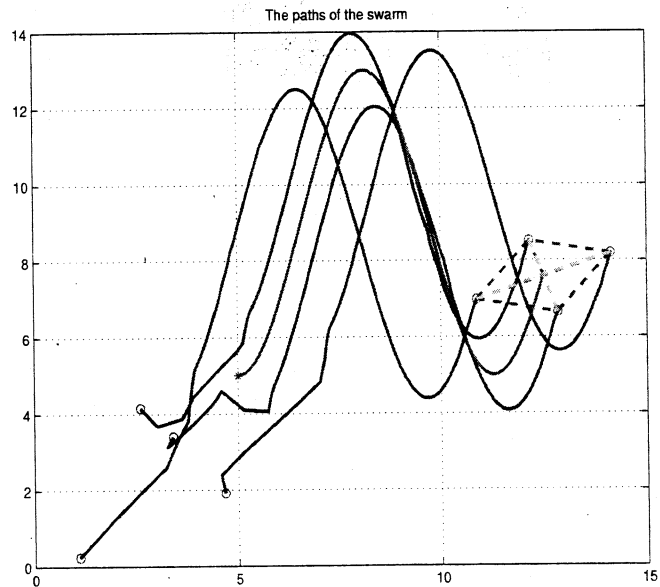


Fig. 1 The paths of the swarm members

$$J(\mathbf{x}, \mathbf{x}_t) = K_T \sum_{i=1}^N \frac{1}{2} (\|\mathbf{x}_i - \mathbf{x}_t\|^2 - \delta_{it}^2)^2 + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{1}{2} (\|\mathbf{x}_i - \mathbf{x}_j\|^2 - \delta_{ij}^2)^2 \right] \quad (26)$$

where δ_{it} and δ_{ij} as defined before.

It is obvious that $J(\mathbf{x}, \mathbf{x}_t) \geq 0$ and has a global minimum at $J(\mathbf{x}, \mathbf{x}_t)=0$, which occurs when $\|\mathbf{x}_i - \mathbf{x}_t\| = \delta_{it}$ for all i and $\|\mathbf{x}_i - \mathbf{x}_j\| = \delta_{ij}$ for all pairs (i, j) . Therefore, in light of the discussions in the preceding sections, we would expect the formation control and the capture of the target to be exactly achieved (which is the case as seen below) provided that the initial conditions are "close enough" to the desired configuration.

For the controller parameters, we use $\alpha=0.01$, $\beta=2.0$, and $\varepsilon=1$. For the filter, we chose $\mu=0.1$. With the proper choice of the agent initial conditions, we get $\bar{J}=0.4$ and $\bar{J}_s=40$, which produce the value of $u_0=124.5$.

For the simulations below, we let $\delta_{it}=1$ and $\sqrt{3}$ as the desired distance between the target and agent, and $\delta_{ij}=2$ for all i and j as the desired distance between agents.

From Fig. 1, we can see that after a short period, the group catches up with the target, and Fig. 2 shows that the target almost stays at the midcenter of the diamond. The tiny error is due to the existence of the filter.

Case 2. For this case, we chose the potential function as

$$J(\mathbf{x}, \mathbf{x}_t) = K_T \sum_{i=1}^N \frac{1}{2} (\|\mathbf{x}_i - \mathbf{x}_t\|^2 - \delta_{it}^2)^2 + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{a_{ij}}{2} \|\mathbf{x}_i - \mathbf{x}_j\|^2 + \frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{c_{ij}}\right) \right] \quad (27)$$

where the parameters a_{ij} , b_{ij} , and c_{ij} depend on the desired relative distances δ_{ij} of the individuals [11]. Here, $a_{ij}/2 \|\mathbf{x}_i - \mathbf{x}_j\|^2$ is the attraction part between agents and $b_{ij} c_{ij}/2 \exp[-(\|\mathbf{x}_i - \mathbf{x}_j\|^2)/c_{ij}]$ is the repulsion part. The distance $\delta_{ij} = \sqrt{c_{ij} \ln(b_{ij}/a_{ij})}$ is the distance at which the attraction and repulsion balance [17,18], which also defines the distance between the agents in the desired formation. We notice that J has a minimum at $J(\mathbf{x}, \mathbf{x}_t)=0$, which occurs when $\|\mathbf{x}_i - \mathbf{x}_t\| = \delta_{it}$ for all i and $\|\mathbf{x}_i - \mathbf{x}_j\| = \delta_{ij}$ for all pairs (i, j) .

For the simulations, we let $\delta_{it}=1$ and $\sqrt{3}$, and $\delta_{ij}=2$. We got the controller parameter with the same procedure as in Case 1. From

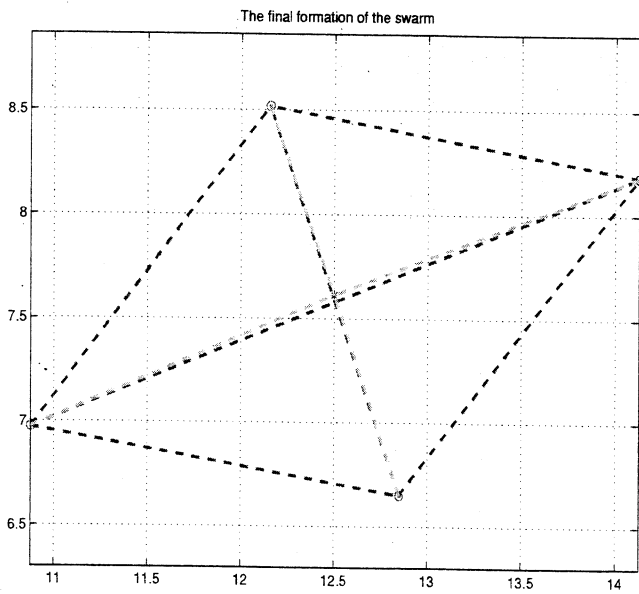


Fig. 2 Zoomed out final position of the swarm

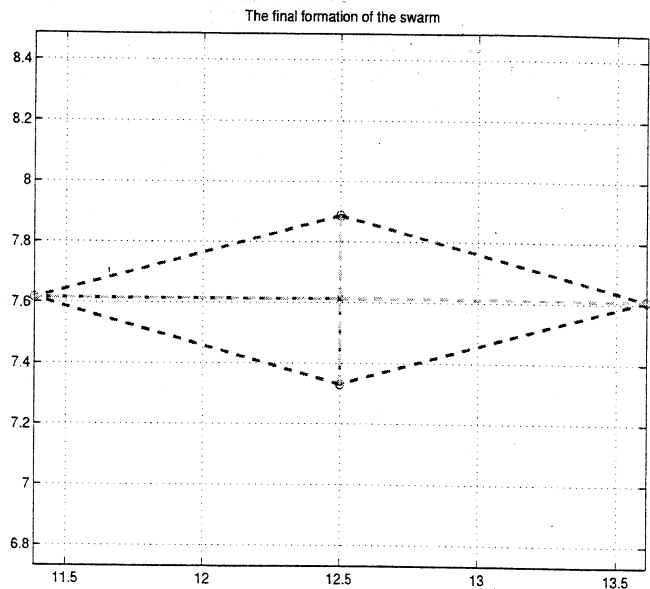


Fig. 4 Zoomed out final position of the swarm

Fig. 3, we see that after a short period, the group follows the target but does not achieve the formation we want. The target even goes out of the group. Here, the system is locked at a local minimum, which corresponds to an undesired configuration.

Now, redefine the potential function as

$$J(\mathbf{x}, \mathbf{x}_t) = K_T \sum_{i=1}^N \frac{1}{3} [(\|\mathbf{x}_i - \mathbf{x}_t\|^2 - \delta_{it}^2)^3 + \delta_{it}^6] + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{a_{ij}}{2} \|\mathbf{x}_i - \mathbf{x}_j\|^2 + \frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{c_{ij}}\right) \right] \quad (28)$$

$J(\mathbf{x}, \mathbf{x}_t) \geq 0$ and $h^u(\|\mathbf{y}\|) = \sum_{i=1}^N (\|\mathbf{y}\|^2 - \delta_{it}^2)^2$ satisfies the condition that $h^u(\|\mathbf{y}\|) > 0$ for all \mathbf{y} except $\|\mathbf{y}\| = \delta_{it}$. Therefore, with the same parameters, we would expect that the agents would at least enclose the target, regardless of initial conditions. From Fig. 4, we can see that after a short period, the group tracks and encloses the target.

It is well known that, practically, potential functions may have many local minima. Therefore, usually when potential functions are used for coordinated tracking in a formation, it is impossible to derive with global results, but only local results can be ob-

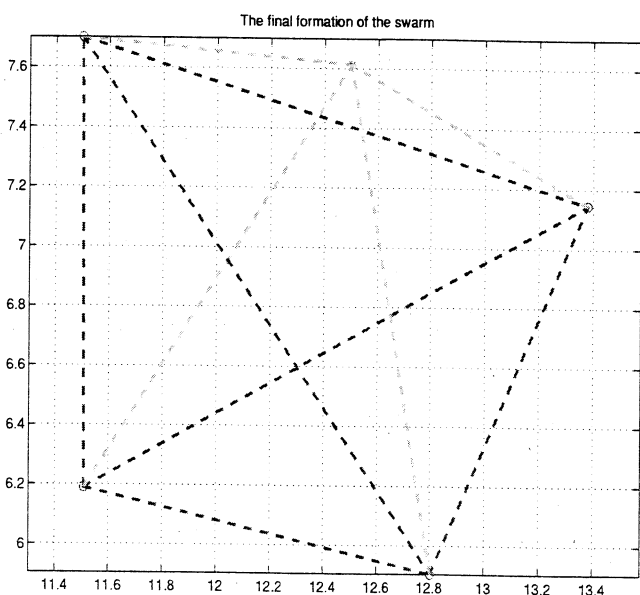


Fig. 3 Zoomed out final position of the swarm

tained. This problem has been observed in many other practical implementations [9]. In particular, for the formation control problem we are considering, there may be several different families of minima. The first family is at the desired formation and the other families occur when the agents are at points at which the attraction and repulsion forces balance. This requires careful consideration and more research. The beauty of the sliding mode control method discussed here is that it is independent of the potential function chosen as long as the boundedness assumptions are satisfied. Moreover, the objectives are achieved despite the model uncertainties.

5 Conclusions

In this paper we extended the work of [12] to the case of multiple agents tracking a target in a formation. We presented a procedure to implement coordinated tracking problem. Here, the potential function for individual includes both the tracking term and the formation term, so that each agent will react if any given agent cannot keep up with the target. This method maintains a more robust formation than the leader-follower model. We obtained the stability proof based on the potential function in a general form. Sliding mode control technique is used to apply the basic design result to the group with general vehicle agents.

Results were obtained from three different potential functions. Future research may focus on extending the procedure to systems composed of agents with non-holonomic constraints as well as handling the trade-off between robustness and communication bandwidth.

Acknowledgment

This work was supported by AFRL/AFOSR Grant No. F33615-01-2-3154. In addition, this work was supported by Dayton Area Graduate Studies Institute (DAGSI). The work of Veysel Gazi was supported in part by TUBITAK (The Scientific and Technological Research Council of Turkey) under Grant No. 104E170 and by the European Commission under FG6 contract No. 045269.

References

- [1] Balch, T., and Arkin, R. C., 1998, "Behavior-Based Formation Control for Multirobot Teams," *IEEE Trans. Rob. Autom.*, **14**, pp. 926-939.
- [2] Olfati-Saber, R., and Murray, R. M., 2002, "Distributed Cooperative Control of Multiple Vehicle Formations Using Structural Potential Functions," *Proceedings of the IFAC World Congress*, Barcelona, Spain, June.
- [3] Kim, D. H., Wang, H. O., Ye, G., and Shin, S., 2004, "Decentralized Control of Autonomous Swarm Systems Using Artificial Potential Functions: Analytical Design Guidelines," *IEEE Conference on Decision and Control*, Dec. 14-

- 17, Conf. 43, Vol. 1, pp. 159–164.
- [4] Raffard, R. L., Tomlin, C. J., and Boyd, S. P., 2004, "Distributed Optimization for Cooperative Agents: Application to Formation Flight," *IEEE Conference on Decision and Control*, Dec. 14–17, Conf. 43, Vol. 3, pp. 2453–2459.
- [5] Yamaguchi, H., 1999, "A Cooperative Hunting Behavior by Mobile-Robot Troops," *Int. J. Robot. Res.*, **18**(8), pp. 931–940.
- [6] Desai, J. P., Ostrowski, J., and Kumar, V., 1998, "Controlling Formations of Multiple Mobile Robots," *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*, Leuven, Belgium, May 16–20, Vol. 4, pp. 2864–2869.
- [7] Ögren, P., Fiorelli, E., and Leonard, N. E., 2004, "Cooperative Control of Mobile Sensor Networks: Adaptive Gradient Climbing in a Distributed Environment," *IEEE Trans. Autom. Control*, **49**(8), pp. 1292–1302.
- [8] Lawton, J. R., Young, B. J., and Beard, R. W., 2000, "A Decentralized Approach to Elementary Formation Maneuvers," *Proceedings of the IEEE International Conference on Robotics and Automation*, San Francisco, CA, April, Vol. 3, pp. 2728–2733.
- [9] Rimon, E., and Koditschek, D. E., 1992, "Exact Robot Navigation Using Artificial Potential Functions," *IEEE Trans. Rob. Autom.*, **8**(5), pp. 501–518.
- [10] Leonard, N. E., and Fiorelli, E., 2001, "Virtual Leaders, Artificial Potentials and Coordinated Control of Groups," *Proceedings of the 40th IEEE Conference on Decision and Control*, Dec. 4–7, Vol. 3, pp. 2968–2973.
- [11] Gazi, V., 2005, "Swarm Aggregations Using Artificial Potentials and Sliding Mode Control," *IEEE Trans. Rob. Autom.*, **21**(6), pp. 1208–1214.
- [12] Gazi, V., and Ordóñez, R., 2004, "Target Tracking Using Artificial Potentials and Sliding Mode Control," *Proceedings of the American Control Conference*, Boston, MA, June–July, Vol., pp. 5588–5593.
- [13] Yao, J., Ordóñez, R., and Gazi, V., 2006, "Swarm Tracking Using Artificial Potentials and Sliding Mode Control," *Proceedings of the IEEE Conference on Decision and Control Conference*, San Diego, CA, Dec., pp. 4670–4675.
- [14] Gazi, V., and Passino, K. M., 2004, "Stability Analysis of Social Foraging Swarms," *IEEE Trans. Syst., Man, Cybern., Part B: Cybern.*, **34**, pp. 539–557.
- [15] Haskara, İ., Özgüner, Ü., and Utkin, V. I., 1998, "On Sliding Mode Observers via Equivalent Control Approach," *Int. J. Control*, **71**, pp. 1051–1067.
- [16] Egerstedt, M., and Hu, X., 2001, "Formation Constrained Multi-Agent Control," *IEEE Trans. Rob. Autom.*, **17**(6), pp. 947–951.
- [17] Gazi, V., and Passino, K. M., 2003, "Stability Analysis of Swarms," *IEEE Trans. Autom. Control*, **48**, pp. 692–697.
- [18] Gazi, V., and Passino, K. M., 2002, "A Class of Attractions/Repulsion Functions for Stable Swarm Aggregations," *Int. J. Control*, **77**(18), pp. 1567–1579.